EXERCISE – IV — HINTS & SOLUTIONS

Sol.1 $\sin 5x = \cos 2x$

$\Rightarrow \sin 5x - \sin\left(\frac{\pi}{2} - 2x\right) = 0$

$\Rightarrow \cos\left(\frac{3x + \frac{\pi}{2}}{2}\right) \sin\left(\frac{7x - \frac{\pi}{2}}{2}\right) = 0$

$\Rightarrow \cos\left(\frac{3x + \frac{\pi}{2}}{2}\right) = 0$ or $\sin\left(\frac{7x - \frac{\pi}{2}}{2}\right) = 0$

$3x + \frac{\pi}{2} = (2n + 1)\pi,\ n \in I$ or $7x - \frac{\pi}{2} = 2n\pi,\ n \in I$

$3x = 2n\pi + \frac{\pi}{2},\ n \in I$ or $7x = 2n\pi + \frac{\pi}{2}$

$x = (4n + 1)\frac{\pi}{6},\ n \in I$ or $x = (4n + 1)\frac{\pi}{14}$

$x = 30^\circ, 150^\circ$ $x = \frac{90^\circ}{7}, \frac{450^\circ}{7}, \frac{810^\circ}{7}, \frac{1170^\circ}{7}$

Sol.2 $\log_{\frac{-x^2-6x}{10}} (\sin 3x + \sin x) = \log_{\frac{-x^2-6x}{10}} (\sin 2x)$

$\Rightarrow \sin 3x + \sin x = \sin 2x$

$\Rightarrow 2 \sin 2x \cos x - \sin 2x = 0$

$\Rightarrow 2 \cos x - 1 = 0 \qquad \because \sin 2x > 0$

$\Rightarrow \cos x = \frac{1}{2} \Rightarrow x = 2n\pi \pm \frac{\pi}{3}\ n \in I \quad ...(i)$

$\because \frac{-x^2 - 6x}{10} > 0 \Rightarrow x(x + 6) < 0 \Rightarrow x \in (-6, 0)$

so by (i) , $x = \left\{\frac{-5\pi}{3}, \frac{-4\pi}{3}, -\pi, \frac{-2\pi}{3}, \frac{-\pi}{3}\right\}$

but $\sin 2x > 0 \Rightarrow x = \left\{-\frac{5\pi}{3}, -\frac{2\pi}{3}\right\}$

& $\sin 3x + \sin x > 0 \Rightarrow 3 \sin x - 4\sin^3 x + \sin x > 0$

$\Rightarrow 4 \sin x (1 - \sin^2 x) > 0 \Rightarrow 4 \sin x \cos^2 x > 0$

$\Rightarrow \sin > 0 \qquad$ Only when $x = \frac{-5\pi}{3}$

$\sin x < 0$ if $x = \frac{-2\pi}{3} \qquad \therefore x = -\frac{5\pi}{3}$

Sol.3 $3 - 2 \cos\theta - 4 \sin\theta - \cos 2\theta + \sin 2\theta = 0$

$\Rightarrow 2 - 2 \cos\theta - 2 \sin\theta + \sin 2\theta - 2 \sin\theta + (1 - \cos 2\theta) = 0$

$\Rightarrow 2(1 - \cos\theta)(1 - \sin\theta) - 2 \sin\theta (1 - \sin\theta) = 0$

$\Rightarrow (1 - \sin\theta)[1 - \cos\theta - \sin\theta] = 0$

$\Rightarrow \sin\theta = 1$ or $\sin\theta + \cos\theta = 1$

$\theta = 2n\pi + \frac{\pi}{2},\ n \in I$ or $\cos\left(\theta - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

or $\theta - \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4}$

or $\theta = 2n\pi + \frac{\pi}{2}$ or $\theta = 2n\pi$

$\therefore x \in \theta = 2n\pi,\ 2n\pi + \frac{\pi}{2},\ n \in I$

Sol.4 $\sin\pi x + \cos\pi x = 0 \qquad \because \qquad x \in [0, 100]$

$\Rightarrow \cos\left(\pi x - \frac{\pi}{4}\right) = 0$

$\pi x - \frac{\pi}{4} = (2n + 1)\frac{\pi}{2}$

$x = n + \frac{1}{2} + \frac{1}{4}$

$x = n + \frac{3}{4}\ n \in I$ or $n - \frac{1}{4}\ n \in I$

$\text{sum} = \left(0 + \frac{3}{4}\right) + \left(1 + \frac{3}{4}\right) + \left(2 + \frac{3}{4}\right) + ... + \left(99 + \frac{3}{4}\right)$

$= (1 + 2 + 3 + 99) + \frac{3}{4} \times 100$

$= \frac{99 \times 100}{2} + 75 = 4950 + 75 = 5025$

Sol.5 $2(\cos x + \cos 2x) + \sin 2x (1 + 2\cos x) = 2\sin x,\ x \in [-\pi, \pi]$

$\Rightarrow 2(\cos x + \cos 2x) + 2 \sin x \cos x (1 + 2\cos x) - 2 \sin x = 0$

$\Rightarrow 2(\cos x + \cos 2x) + 2\sin x\,[\cos x + (2\cos^2 x - 1)] = 0$

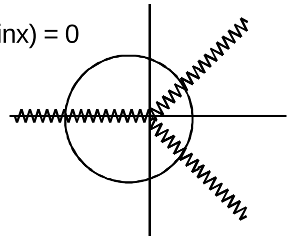
$\Rightarrow 2\,(\cos x + \cos 2x)\,(1 + \sin x) = 0$

$\Rightarrow 2^2 \cos\frac{3x}{2}\,\cos\frac{x}{2}\,(1 + \sin x) = 0$

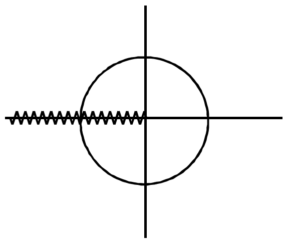
$\Rightarrow \cos\frac{3x}{2} = 0$

or $\cos\frac{x}{2} = 0$

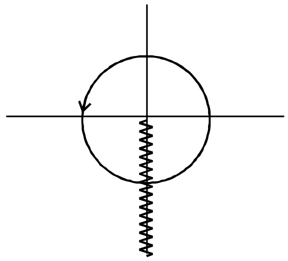
or $\sin x = -1$



$x = -\pi, -\frac{\pi}{3}, \frac{\pi}{3}, \pi$



$x = \pm\pi$



$x = -\frac{\pi}{2}$

$\Rightarrow \quad x = \pm\frac{\pi}{3}, -\frac{\pi}{2}, \pm\pi$

Sol.6 $y + \cos x = \sin x$

$\Rightarrow \sin x - \cos x = y \Rightarrow -\sqrt{2} \le \sin x - \cos x \le \sqrt{2}$

$\Rightarrow -\sqrt{2} \le y \le \sqrt{2}$

$y = 1 \quad \sin\left(x - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

$x - \frac{\pi}{4} = \frac{\pi}{4}, \frac{3\pi}{4} \Rightarrow x = \frac{\pi}{2}, \pi$

Sol.7 $(\sin\theta)\,x^2 + (2\cos\theta)\,x + \left(\frac{\cos\theta + \sin\theta}{2}\right)$

$b^2 - 4ac = 0$

$4\cos^2\theta - 4\sin\theta\left(\frac{\cos\theta + \sin\theta}{2}\right) = 0$

$\Rightarrow 2\cos^2\theta - \sin\theta\,(\cos\theta + \sin\theta) = 0$

$\Rightarrow 2\cos^2\theta - \sin\theta\cos\theta - \sin^2\theta = 0$

$\Rightarrow 2\cos\theta\,(\cos\theta - \sin\theta) + \sin\theta\,(\cos\theta - \sin\theta) = 0$

$\Rightarrow (\cos\theta - \sin\theta)\,(2\cos\theta + \sin\theta) = 0$

$\Rightarrow \cos\theta = \cos\left(\frac{\pi}{2} - \theta\right)$ or $\frac{2}{\sqrt{5}}\cos\theta + \frac{1}{\sqrt{5}}\sin\theta = 0$

$\theta = 2n\pi \pm \left(\frac{\pi}{2} - \theta\right) \qquad \sin(\theta + \alpha) = 0 \;\; \{\tan\alpha = 2\}$

$\theta = 2n\pi \pm \frac{\pi}{2} - \theta \qquad \theta + \alpha = 2n\pi$

$\theta = n\pi \pm \frac{\pi}{4}$

$\sin\theta = \cos\theta = \pm\frac{1}{\sqrt{2}}$

$f(\theta) = \pm\frac{1}{\sqrt{2}}\,x^2 + \left(\frac{\pm 2}{\sqrt{2}}\right)x + \left(\frac{\pm 1}{\sqrt{2}}\right)$

$= \pm\frac{1}{\sqrt{2}}\,(x + 1)^2$

with (–) sign not in square of linear

$= +\frac{1}{\sqrt{2}}\,(x + 1)^2 \qquad \{q\ ^1\ (2n + 1)x + \frac{\pi}{4}\}$

$= \left(\frac{x + 1}{2^{1/4}}\right)^2$

$\theta = n\pi + \frac{\pi}{4}$

$\theta = 2n\pi - \tan^{-1}2$

$\tan(\pi - \alpha) = \tan\alpha = 2$

$\pi - \alpha = \tan^{-1}2$

$\alpha = \pi - \tan^{-1}2$

$\theta = (2n + 1)\,\pi - \tan^{-1}2$

Sol.8 $\tan^2(x + y) + \cot^2(x + y) = 1 - 2x - x^2$

L.H.S. $= \tan^2(x + y) + \frac{1}{\tan^2(x + y)}$

min value is = 2

L.H.S. ≥ 2

R.H.S. = max. value $= -\frac{D}{4a} = \frac{-(4 + 4)}{4(-1)} = 2$

R.H.S. ≤ 2

$\therefore$ L.H.S. = R.H.S. = 2

$\therefore \quad \tan^2(x + y) + \cot^2(x + y) = 2$

$\Rightarrow \quad \tan(x + y) = \cot(x + y)$

$\Rightarrow \tan^2(x + y) = 1 + \tan^2\left(\frac{\pi}{4}\right)$

$\Rightarrow (x + y) = n\pi \pm \frac{\pi}{4} \Rightarrow y = n\pi \pm \frac{\pi}{4} + 1,$

$n \in I$

& $1 - 2x - x^2 = 2 \Rightarrow x^2 + 2x + 1 = 0$

$\Rightarrow (x + 1)^2 = 0 \Rightarrow x = -1$

Sol.9 sin x . sin 2x . sin 3x = 1

Let $0 < \sin x < 1 \Rightarrow \sin 2x \sin 3x > 1$

But it is not possible

$\Rightarrow$ sin x = 1 & sin 2x = 1 & sin 3x = 1

But It's not possible $\Rightarrow$ no sol.

Sol.10 $\sec 4\theta - \sec 2\theta = 2$

$\Rightarrow \cos 2\theta - \cos 4\theta = 2\cos 2\theta \cos 4\theta$

$\Rightarrow \cos 2\theta - \cos 4\theta = \cos 6\theta + \cos 2\theta$

$\Rightarrow \cos 6\theta \quad + \cos 4\theta = 0$

$\Rightarrow 2\cos 5\theta \cos\theta = 0$

$\Rightarrow \cos 5\theta = 0 \quad \text{or} \quad \cos\theta = 0$

$5\theta = (2n+1)\frac{\pi}{2}, n \in I \quad \theta = (2x+1)\frac{\pi}{2}, n \in I$

or $5\theta = 2n\pi \pm \frac{\pi}{2}, n \in I$ or $\theta = 2n\pi \pm \frac{\pi}{2}\ n \in I$

$\theta = \frac{2n\pi}{5} \pm \frac{\pi}{10}, n \in I$ or $\theta = 2n\pi \pm \frac{\pi}{2}\ n \in I$

$\therefore$ sec 4θ & sec 2θ defined at that angles.

Sol.11 $f(x) = \sin^6 x + \cos^6 x + k(\sin^4 x + \cos^4 x)$

$= (\sin^2 x + \cos^2 x)^3 - 3\sin^2 x\cos^2 x(\sin^2 x + \cos^2 x)$

$\quad + k[(\sin^2 x + \cos^2 x)^2 - 2\cos^2 x\sin^2 x]$

$= 1 - 3\sin^2 x\cos^2 x + k - 2k\sin^2 x\cos^2 x$

$f(x) = (k+1) - (2k+3)\sin^2 x\cos^2 x$

(a) f(x) is constant $\forall x \in R$

$\Rightarrow 2k + 3 = 0 \Rightarrow k = \frac{-3}{2}$

(b) f(c) = 0

$\sin^2 x \cos^2 x = \frac{k+1}{2k+3}$

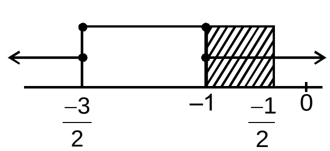
$\Rightarrow 0 \le \sin^2 2x = \frac{4(k+1)}{(2k+3)} \le 1$

$\frac{4(k+1)}{(2k+3)} \ge 0 \quad \& \quad \frac{4(k+1)}{2k+1} \le 1$

$\Rightarrow k \in \left(-\infty, \frac{-3}{2}\right) \cup [-1, \infty)$

& $\quad \frac{4k+4-2k-3}{2k+3} \le 0$

$\frac{2k+1}{2k+3} \le 0$



$k \in \left(-\frac{3}{2}, \frac{-1}{2}\right]$

(c) k = –0.7 f(x) = 0

$\sin^2 2x = \frac{4(-0.7+1)}{2(-0.7)+3} = \frac{4\times 0{,}3}{1{,}6} = \frac{3}{4}$

$\sin^2 2x = \left(\frac{\sqrt{3}}{2}\right)^2$

$2x = n\pi \pm \frac{\pi}{3}, n \in I$

$x = \frac{n\pi}{2} \pm \frac{\pi}{6}$

Sol.12 $\frac{\sqrt{3}}{2}\sin x - \cos x = \cos^2 x$

$\Rightarrow \sqrt{3}\sin x = 2(\cos x + \cos^2 x)$

$\Rightarrow 3\sin^2 x = 4(\cos^2 x + \cos^4 x + 2\cos^3 x)$

squaring

$\Rightarrow 3 - 3\cos^2 x = 4\cos^2 x + 4\cos^4 x + 8\cos^3 x$

$\Rightarrow$ Let cos x = t

$\Rightarrow 4t^4 + 8t^3 + 7t^2 - 3 = 0$

$\Rightarrow (t+1)(4t^3 + 4t^2 + 3t - 3) = 0$

$\Rightarrow t = -1$ or $t = \frac{1}{2}$ or $2t^2 + 3t + 3 \ne 0 \quad \because D < 0$

$\Rightarrow \cos x = -1 \quad \text{or} \quad \cos x = \frac{1}{2}$

$x = 2n\pi \pm \pi, n \in I \quad \text{or} \quad x = 2n\pi \pm \frac{\pi}{3}, n \in I$

Sol.13 $\cos 3x \cdot \cos^3 x + \sin 3x \cdot \sin^3 x = 0$

$\Rightarrow \cos 3x\left[\frac{\cos 3x + 3\cos x}{4}\right] + \sin 3x\left[\frac{3\sin x - \sin 3x}{4}\right] = 0$

$\Rightarrow \cos^2 3x + 3\cos 3x\cos x$

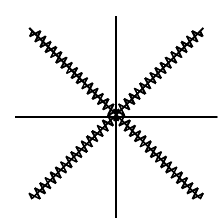
$\quad + 3\sin 3x\sin x - \sin^2 3x = 0$

$\Rightarrow \cos 6x + 3\cos 2x = 0$

$\Rightarrow 4\cos^3 2x - 3\cos 2x + 3\cos 2x = 0$

$\Rightarrow \cos 2x = 0$

$\Rightarrow 2x = 2n\pi \pm \frac{\pi}{2}$



$\Rightarrow x = n\pi \pm \frac{\pi}{4}, n \in I \quad \text{or} \quad x = \frac{n\pi}{2} \pm \frac{\pi}{4}$

Sol.14 $4\cos^2x \sin x - 2\sin^2x = 3\sin x$

$\sin x\,[3 + 2\sin x - 4\cos^2x] = 0$

$\sin x\,[4\sin^2x + 2\sin x - 1] = 0$

$\Rightarrow \sin x = 0 \quad \Rightarrow x = n\pi,\ n \in I$

or $\quad 4\sin^2x + 1\ 2\sin x - 1 = 0$

$$\sin x = \frac{-2 \pm \sqrt{20}}{8} = \frac{-1 \pm \sqrt{5}}{4}$$

$$\sin x = \sin\frac{\pi}{10} \text{ or } \sin x = \sin\left(\frac{-3\pi}{10}\right)$$

$$x = n\pi + (-1)^n\left(\frac{\pi}{10}\right) \text{ or } x = n\pi + (-1)^n\left(\frac{-3\pi}{10}\right)\ n \in I$$

Aliter :

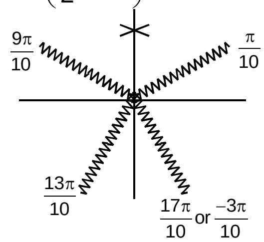
$\sin x\,[(4\cos^2 x - 3) - 2\sin x] = 0$

$$\Rightarrow \sin x\left[\frac{\cos 3x}{\cos x} - 2\sin x\right] = 0$$

$$\Rightarrow \sin x\ \frac{[\cos 3x - \sin 2x]}{\cos x} = 0$$

$$\sin x = 0 \text{ or } \cos 3x = \cos\left(\frac{\pi}{2} - 2x\right)$$

& $\cos x \neq 0$



$x \neq (2n+1)\frac{\pi}{2}$

But

$$x = n\pi\ 5x = 2n\pi + \frac{\pi}{2} \text{ or } x = 2n\pi - \frac{\pi}{2} = (4n-1)\frac{\pi}{2}$$

$$x = \frac{2n\pi}{5} + \frac{\pi}{10} = (4n+1)\frac{\pi}{10}$$

$$\therefore \quad x = n\pi + (-1)^n\frac{\pi}{10}$$

$$x = n\pi + (-1)^n\left(\frac{-3\pi}{10}\right)$$

Sol.15 $a\cos\theta + b\sin\theta = c \begin{cases}\alpha \\ \beta\end{cases}$

$\Rightarrow a\cos\theta = c - b\sin\theta$ squaring

$\Rightarrow a^2\cos^2\theta = c^2 + b^2\sin^2\theta - bc\sin\theta$

$\Rightarrow a^2 - a^2\sin^2\theta = c^2 + b^2\sin^2\theta - 2bc\sin\theta$

$$\Rightarrow (a^2+b^2)\sin^2\theta - 2bc\sin\theta + (c^2 - a^2) = 0 \begin{cases}\sin\alpha \\ \sin\beta\end{cases}$$

(i) $\quad \sin\alpha + \sin\beta = \dfrac{2bc}{a^2+b^2}$

(ii) $\quad \sin\alpha\sin\beta = \dfrac{c^2 - a^2}{a^2+b^2}$

$a\cos\theta + b\sin\theta = c$

$$\Rightarrow \frac{a\left(1 - \tan^2\frac{\theta}{2}\right)}{1 + \tan^2\frac{\theta}{2}} + \frac{2b\tan\frac{\theta}{2}}{1 + \tan^2\frac{\theta}{2}} = c$$

$$\Rightarrow a - a\tan^2\frac{\theta}{2} + 2b\tan\frac{\theta}{2} = c + c\tan^2\frac{\theta}{2}$$

$$\Rightarrow (a+c)\tan^2\frac{\theta}{2} - 2b\tan\frac{\theta}{2} + (c-a) = 0 \begin{cases}\tan\frac{\alpha}{2} \\ \tan\frac{\beta}{2}\end{cases}$$

(iii) $\tan\frac{\alpha}{2} + \tan\frac{\beta}{2} = \dfrac{2b}{a+c}$,

(iv) tran $\frac{\alpha}{2}\ \tan\frac{\beta}{2} = \dfrac{c-a}{c+a}$

Sol.16 $2(\sin x - \cos 2x) - \sin 2x\,(1 + 2\sin x) + 2\cos x = 0$

$\Rightarrow 2(\sin x - \cos 2x) - 2\sin x\cos x\,(1 + 2\sin x) + 2\cos x = 0$

$\Rightarrow 2(\sin x - \cos 2x) - 2\cos x\,[\sin x + (2\sin^2x - 1)] = 0$

$\Rightarrow 2(\sin x - \cos 2x) - 2\cos x\,[\sin x - \cos 2x] = 0$

$\Rightarrow 2(\sin x - \cos 2x)(1 - \cos x) = 0$

$\Rightarrow \cos x = 1 \ \Rightarrow \quad x = 2n\pi,\quad n \in I$

or $\quad \sin x - \cos 2x = 0$

$2\sin^2x + \sin x - 1 = 0$

$(\sin x + 1)(2\sin x - 1) = 0$

$\sin x = -1 \qquad$ or $\qquad \sin x = \dfrac{1}{2}$

$$x = n\pi + (-1)^n\left(-\frac{\pi}{2}\right),\ n \in I$$

or $\ x = n\pi + (-1)^n\left(\dfrac{\pi}{6}\right)\ n \in I$

Sol.17 $\sin 2x > \sqrt{2}\ \sin^2x + (2 - \sqrt{2})\cos^2x$

$2\sin x\cos x > \sqrt{2}\sin^2x + (2 - \sqrt{2})\cos^2x$

divided by $\sqrt{2}\cos^2x \quad \because \quad \cos^2x > 0$

$\{\because \cos x \neq 0$, If $\cos x = 0 \Rightarrow 0 > \sqrt{2}\sin^2x$ which is false.$\}$

$\therefore\ \sqrt{2}\tan x > \tan^2x + (\sqrt{2} - 1)$

$\Rightarrow \tan^2x - \sqrt{2}\tan x + (\sqrt{2} - 1) < 0$

$\Rightarrow (\tan x - 1)\,[\tan x - (\sqrt{2} - 1)] < 0$

$(\sqrt{2} - 1) < \tan x < 1$

$\Rightarrow n\pi + \frac{\pi}{8} < x < n\pi + \frac{\pi}{4} \quad n \in I$

Sol.18 $\cos 3x + \cos 2x = \sin\frac{3x}{2} + \sin\frac{x}{2}$, $x \in (0, 2\pi)$

$\Rightarrow 2\cos\frac{5x}{2}\,.\,\cos\frac{x}{2} = 2\sin x\cos\frac{x}{2}$

$\Rightarrow 2\cos\frac{x}{2}\left[\cos\frac{5x}{2} - \sin x\right] = 0$

$\Rightarrow \cos\frac{x}{2} = 0$ or $\cos\frac{5x}{2} = \cos\left(\frac{\pi}{2} - x\right)$

$\Rightarrow \frac{x}{2} = \frac{\pi}{2}, \frac{3\pi}{2} \quad \therefore x = \pi$

$\frac{5x}{2} = 2n\pi + \frac{\pi}{2} - x$ or $\frac{5x}{2} = 2n\pi - \frac{\pi}{2} + x$

$\frac{7x}{2} = (4n + 1)\frac{\pi}{2}$ or $\frac{3x}{2} = (4n - 1)\frac{\pi}{2}$

$x = (4n + 1)\frac{\pi}{7}$ $\qquad$ $x = (4n - 1)\frac{\pi}{3}$

$x = \frac{\pi}{7}, \frac{5\pi}{7}, \frac{9\pi}{7}, \frac{13\pi}{7}$ $\qquad$ $x = -\frac{\pi}{3}, \pi, \frac{7\pi}{3}$

$\frac{7\pi}{3} > 2\pi$

Sol.19 $\sin 3\alpha = 4\sin\alpha\sin(x + \alpha)\sin(x - \alpha)$, α is constant.

$\sin\alpha\,(3 - 4\sin^2\alpha) = 4\sin\alpha\,[\sin^2x - \sin^2\alpha]$

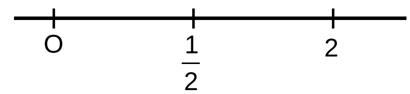
$\Rightarrow 3 - 4\sin^2\alpha = 4\sin^2x - 4\sin^2\alpha$

$\Rightarrow \sin^2x = \left(\frac{\sqrt{3}}{2}\right)^2 = \sin^2\frac{\pi}{3}$

$x = n\pi \pm \frac{\pi}{3} \qquad n \in I$

Sol.20 $|2\tan x - 1| + |2\cot x - 1| = 2$

points $\tan x = \frac{1}{2}$ & $\cot x = \frac{1}{2}$ or $\tan x = 2$



Case - I : $\tan x \ge 2$

$2\tan x - 1 - \frac{2}{\tan x} + 1 = 2$

$\Rightarrow \tan x - \frac{1}{\tan x} = 1 \quad \Rightarrow \tan^2x - \tan x - 1 = 0$

$\Rightarrow \tan x = -\frac{1 \pm \sqrt{5}}{2} \notin [2, \infty)$ (reject)

Case-II : $\frac{1}{2} \le \tan < 2$

$2\tan x - 1 + \frac{2}{\tan x} - 1 = 2$

$\Rightarrow \tan x + \frac{1}{\tan x} = 2 \quad \Rightarrow \tan x = 1 \Rightarrow x = \frac{\pi}{4}$

Case III : $0 \le \tan x < \frac{1}{2}$

$-2\tan x + 1 + \frac{2}{\tan x} - 1 = 2$

$\Rightarrow -\tan x + \frac{1}{\tan x} = 1$

$\Rightarrow \tan^2x + \tan x - 1 = 0$

$\Rightarrow \tan x = \frac{-1 \pm \sqrt{5}}{2}$ (–) sign reject

$\tan x = \frac{\sqrt{5} - 1}{2} > \frac{1}{2}$, reject

Case IV : $\tan x < 0$

$-2\tan x + 1 - \frac{2}{\tan x} + 1 = 2$

$\Rightarrow \tan x + \frac{1}{\tan x} = 0$ $\qquad$ not possible

$\therefore \tan x = 1 \qquad \Rightarrow x = n\pi + \frac{\pi}{4}$, $n \in I$

Sol.21 $\sin^4x + \cos^4x + \sin 2x + a = 0$

$\Rightarrow (1)^2 - 2\sin^2x\cos^2x + \sin 2x + a = 0$

$\Rightarrow 1 - \dfrac{\sin^2 2x}{2} + \sin 2x + a = 0$

$\Rightarrow \sin^2 2x - 2\sin 2x - 2(a+1) = 0$

$\Rightarrow (\sin 2x - 1)^2 = 2a + 3$

$\because\ 0 \le (\sin 2x - 1)^2 \le (2)^2 \ \Rightarrow\ 0 \le 2a + 3 \le 4$

$\Rightarrow\ \dfrac{-3}{2} \le a \le \dfrac{1}{2} \Rightarrow a \in \left[\dfrac{-3}{2}, \dfrac{1}{2}\right]$

$\sin 2x - 1 = \pm\sqrt{2a+3}$

$\sin 2x = 1 \pm \sqrt{2a+3}$ (+) will not be consider

$\because\quad -1 \le \sin 2x \le 1$

$\sin 2x = 1 - \sqrt{2a+3}$

$2x = n\pi + (-1)^n \sin^{-1}(1 - \sqrt{2a+3})$

$x = \dfrac{1}{2}[n\pi + (-1)^n \sin^{-1}(1 - \sqrt{2a+3})]$

Sol.22 $\tan^2 2x + \cot^2 2x + 2\tan 2x + 2\cot 2x = 6$

$\Rightarrow \left(\tan^2 2x + \dfrac{1}{\tan^2 2x}\right) + 2\left(\tan 2x + \dfrac{1}{\tan 2x}\right) = 6$

$\Rightarrow \left(\tan 2x + \dfrac{1}{\tan 2x}\right)^2 - 2 + 2\left(\tan 2x + \dfrac{1}{\tan 2x}\right) - 6 = 0$

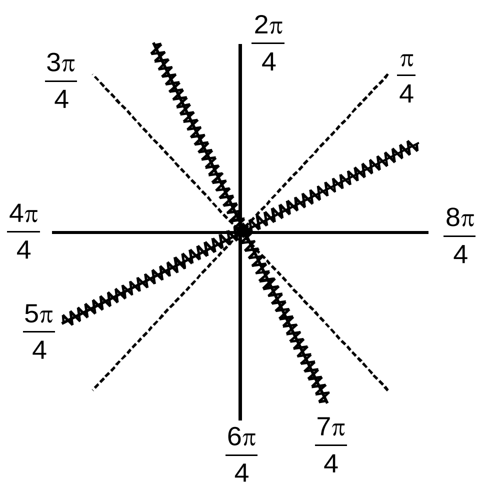
$\Rightarrow\quad t^2 + 2t - 8 = 0 \ \Rightarrow\ (t+4)(t-2) = 0$

(where $t = \tan 2x + 1/\tan 2x$)

$\therefore \tan 2x + \dfrac{1}{\tan 2x} = 2 \ \Rightarrow\ (\tan 2x - 1)^2 = 0$

$\Rightarrow\quad \tan 2x = 1 \ \Rightarrow\ 2x = n\pi + \dfrac{\pi}{4}, n \in I$

$\Rightarrow\quad x = \dfrac{n\pi}{2} + \dfrac{\pi}{8}, n \in I$



or $\quad x = \dfrac{n\pi}{4} + (-1)^n \dfrac{\pi}{8}$

or $\tan 2x + \dfrac{1}{\tan 2x} = -4$

$\tan^2 2x + 4\tan 2x + 1 = 0$

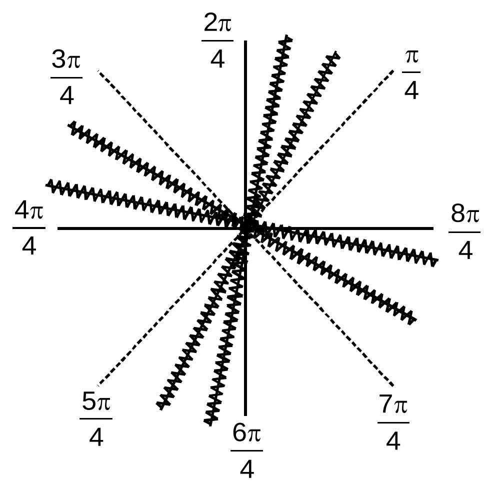
$(\tan 2x + 2)^2 = 3$

$\tan 2x = -(2 - \sqrt{3})$ or $\tan 2x = -(2 + \sqrt{3})$

$= \tan\left(-\dfrac{\pi}{12}\right) \qquad = \tan\left(\dfrac{-5\pi}{12}\right)$

$2x = n\pi - \dfrac{\pi}{12}$ or $2x = n\pi - \dfrac{5\pi}{12}$

$x = \dfrac{n\pi}{2} - \dfrac{\pi}{12}$ or $x = \dfrac{n\pi}{2} - \dfrac{5\pi}{24}$



$n \in I$

$x = \dfrac{n\pi}{4} + (-1)^{n+1}\dfrac{\pi}{24}$

Sol.23 $\sin^4x + \cos^4x - 2\sin^2x + \dfrac{3}{4}\sin^2 2x = 0$

$\Rightarrow\quad (\sin^2x + \cos^x)^2 - 2\sin^2x\cos^2x - 2\sin^2x + 3\sin^2x\cos^2x = 0$

$\Rightarrow\quad 1 + \sin^2x\cos^2x - 2\sin^2x = 0$

$\Rightarrow\quad 1 + \sin^2x - \sin^4x - 2\sin^2x = 0$

$\Rightarrow\quad \sin^4x + \sin^2x - 1 = 0$

$\Rightarrow\quad \sin^2x - \dfrac{-1 \pm \sqrt{1+4}}{2} = \dfrac{-1 \pm \sqrt{5}}{2}$

$\sin^2x > 0, \quad \therefore \quad$ (–) reject

$\therefore\quad \sin^2x = \dfrac{\sqrt{5} - 1}{2}$

$2\sin^2 x = \sqrt{5} - 1$

$\Rightarrow \quad 1 - \cos 2x = \sqrt{5} - 1$

$\Rightarrow \quad \cos 2x = 2 - \sqrt{5} = \cos \alpha$

$2x = 2n\pi \pm \alpha \quad \{\alpha = \cos^{-1}(2 - \sqrt{5})\}$

$x = n\pi \pm \dfrac{1}{2}\cos^{-1}(2 - \sqrt{5})\ n \in I$

Sol.24 $\tan^2 x \cdot \tan^2 3x \cdot \tan 4x = \tan^2 x - \tan^2 3x + \tan 4x$

$\Rightarrow \tan^2 3x - \tan^2 x = \tan 4x\,(1 - \tan^2 3x \tan^2 x)$

$\Rightarrow \tan 4x = \dfrac{(\tan 3x + \tan x)}{(1 - \tan 3x \tan x)} \cdot \dfrac{(\tan 3x - \tan x)}{(1 + \tan 3x \tan x)}$

$\Rightarrow \tan 4x = \tan 4x \cdot \tan 2x$

$\Rightarrow \tan 4x\,(\tan 2x - 1) = 0$

$\Rightarrow \tan 4x = 0$ or $\tan 2x = 1$

$\Rightarrow 4x = n\pi$ or $2x = n\pi + \dfrac{\pi}{4} = (4n+1)\dfrac{\pi}{4}$

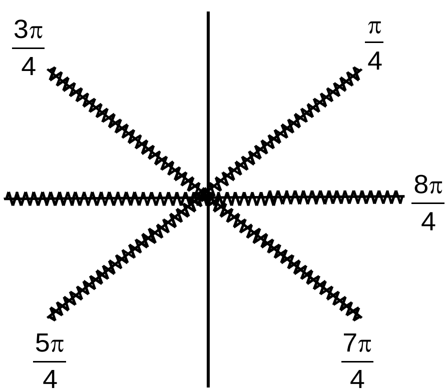
$\Rightarrow x = \dfrac{n\pi}{4}\ .\ n \in I$ or $x = (4n+1)\dfrac{\pi}{8}\, n \in I$

$4x = (4n+1)\dfrac{\pi}{2}$

but $x \neq (2n+1)\dfrac{\pi}{2}$ $\quad \tan 4x =$ N.D.

$\tan x =$ N.D. $\quad x \neq (4n+1)\dfrac{\pi}{8}$

$\therefore$



$x = n\pi\ \ n \in I$ or $x = n\pi \pm \dfrac{\pi}{4}$ or $x = (2n+1)\dfrac{\pi}{4}$

Sol.25 $\sin^{10} x + \cos^{10} x = \dfrac{29}{16}\cos^4 2x$

$\Rightarrow \left(\dfrac{1-\cos 2x}{2}\right)^5 + \left(\dfrac{1+\cos 2x}{2}\right)^5 = \dfrac{29}{16}\cos^4 2x$

Let $\cos 2x = t$

$\Rightarrow (1-t)^t + (1+t)^5 = (29) \cdot 2t^4$

$\Rightarrow 1 + 5t + 10t^2 + 10t^3 + 5t^4 + t^5$

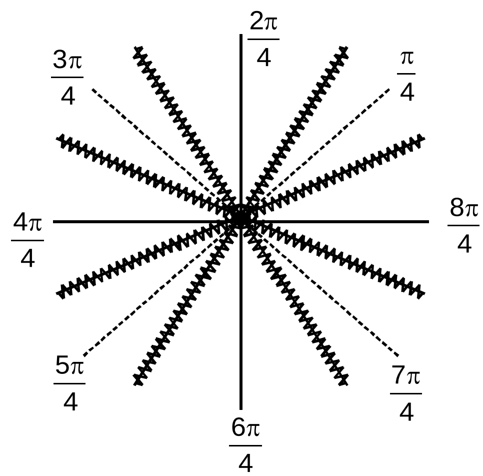
$+1 - 5t + 10t^2 - 10t^3 + 5t^4 - t^5 = (29)\, 2t^4$

$\Rightarrow 2(1 + 10t^2 + 5t^4) = 2.\ (29)\ t^4$

$\Rightarrow 24t^4 - 10t^2 - 1 = 0 \Rightarrow (2t^2 - 1)(12t^2 + 1) = 0$

$\Rightarrow t^2 = \dfrac{1}{2}\ \cos^2 2x = \dfrac{1}{2} = \left(\dfrac{1}{\sqrt{2}}\right)^2\ (\because 12t^2 + 1 \neq 0)$

$\Rightarrow 2x = n\pi \pm \dfrac{\pi}{4} \Rightarrow x = \dfrac{n\pi}{2} \pm \dfrac{\pi}{8},\quad n \in I$



or $\quad x = \dfrac{n\pi}{4} + \dfrac{\pi}{8},\ n \in I$

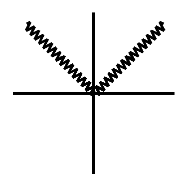
Sol.26 $\sin\left(x - \dfrac{\pi}{4}\right) - \cos\left(x + \dfrac{3\pi}{4}\right) = 1$

$\Rightarrow \quad \sin\left(x - \dfrac{\pi}{4}\right) - \cos\left(\dfrac{\pi}{2} + x + \dfrac{\pi}{4}\right) = 1$

$\Rightarrow \quad \sin\left(x - \dfrac{\pi}{4}\right) + \sin\left(x + \dfrac{\pi}{4}\right) = 1$

$\Rightarrow \quad 2\sin x \cos\dfrac{\pi}{4} = 1 \quad \Rightarrow \quad \dfrac{2}{\sqrt{2}}\sin x = 1$

$\Rightarrow \quad \sin x = \dfrac{1}{\sqrt{2}}$



$x = 2n\pi + \frac{\pi}{4}$ or $x = 2n\pi + \frac{3\pi}{4}$

Check $x = \frac{\pi}{4}$

$\Rightarrow \frac{2\cos 7x}{\cos 3 + \sin 3} > 2^{\cos 2x} \Rightarrow \frac{2\frac{1}{\sqrt{2}}}{(-ve)} > 2^{o}$ wrong

Check $x = \frac{3\pi}{4}$

$\Rightarrow \frac{2\cos\frac{21\pi}{4}}{(-ve)} > 2^{\cos\frac{6\pi}{4}} \Rightarrow \frac{2\left(-\frac{1}{\sqrt{2}}\right)}{(-ve)} > 2^{o}$ True

$\therefore\ x = 2n\pi + \frac{3\pi}{4},\ n \in I$

Sol.27 $\sin\sqrt{x} = -1 \qquad x > 0$

$= -\sin\frac{\pi}{2} = \sin\frac{3\pi}{2} \qquad x \neq -\frac{\pi}{2}$

$\sqrt{x} = 2n\pi + \frac{3\pi}{2} \qquad n \in I$

$\sqrt{x} = \frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}, \frac{15\pi}{2}, \frac{19\pi}{2}, \frac{23\pi}{2}, ..$

$x = \frac{9\pi^2}{4}, \frac{49\pi^2}{4}, \frac{121\pi^2}{4}, \frac{225\pi^2}{4}, \frac{361\pi^2}{4}, ... \frac{529\pi^2}{4}, ...$

Sum of root which are less than $100\pi^2$

$= \frac{9\pi^2}{4} + \frac{49\pi^2}{4} + \frac{121\pi^2}{4} + \frac{225\pi^2}{4} + \frac{361\pi^2}{4} = \frac{765\pi^2}{4}$

Sum of square root of these roots

$= \frac{3\pi}{2} + \frac{7\pi}{2} + \frac{11\pi}{2}, \frac{15\pi}{2}, \frac{19\pi}{2} = \frac{55\pi}{2}$

Now, $\cos\sqrt{x} = 0 \qquad x > 0$

$\Rightarrow \cos\sqrt{x} = \cos\frac{\pi}{2}$

$\Rightarrow \sqrt{x} = 2n\pi + \frac{\pi}{2}$ or $\sqrt{x} = 2n\pi + \frac{3\pi}{2}$

$\Rightarrow x = \left(2n\pi + \frac{\pi}{2}\right)^2 \qquad x = \left(2n\pi + \frac{2\pi}{2}\right)^2$

So we can conclude the all the roots of $\cos\sqrt{x} = 0$ are not also the roots of $\sin\sqrt{x} = -1$

Sol.28 $\sin\left(\frac{\sqrt{x}}{2}\right) + \cos\left(\frac{\sqrt{x}}{2}\right) = \sqrt{2}\ \sin\sqrt{x} \qquad x > 0$

$\Rightarrow \frac{1}{\sqrt{2}}\sin\left(\frac{\sqrt{x}}{2}\right) + \frac{1}{\sqrt{2}}\cos\left(\frac{\sqrt{x}}{2}\right) = \sin\sqrt{x}$

$\Rightarrow \sin\left(\frac{\sqrt{x}}{2} + \frac{\pi}{4}\right) = \sin\sqrt{x}$

$\Rightarrow \frac{\sqrt{x}}{2} + \frac{\pi}{4} = 2n\pi + \sqrt{x}$

$\Rightarrow -\frac{\sqrt{x}}{2} = 2n\pi - \frac{\pi}{4}$

$\Rightarrow x = \left(4n\pi - \frac{\pi}{2}\right)^2,\ n \in I$

or $\frac{\sqrt{x}}{2} + \frac{\pi}{4} = 2n\pi + \pi - \sqrt{x}$

$\Rightarrow \frac{3\sqrt{x}}{2} = 2n\pi + \frac{3\pi}{4} \Rightarrow \sqrt{x} = \frac{4n\pi}{3} + \frac{\pi}{2}$

$\Rightarrow x = \left(\frac{4n\pi}{3} + \frac{\pi}{2}\right)^2\ n \in I$

Sol.29 $\sin\left(\frac{2x+1}{x}\right) + \sin\left(\frac{2x+1}{3x}\right) - 3\cos^2\left(\frac{2x+1}{3x}\right) = 0$

$\Rightarrow \left\{\sin\theta + \sin\frac{\theta}{3}\right\} - 3\cos^2\frac{\theta}{3} = 0 \quad \left(\text{Let } \frac{2x+1}{x} = \theta\right)$

$\Rightarrow 2\sin\frac{2\theta}{3}\cos\frac{\theta}{3} - 3\cos^2\frac{\theta}{3} = 0$

$\Rightarrow 2.2\sin\frac{\theta}{3}\cos^2\frac{\theta}{3} - 3\cos^2\frac{\theta}{3} = 0$

$\Rightarrow \cos^2\frac{\theta}{3}\left[4\sin\frac{\theta}{3} - 3\right] = 0 \Rightarrow \cos^2\left(\frac{2x+1}{3x}\right) = 0$

$\Rightarrow \frac{2x+1}{3x} = (2n+1)\frac{\pi}{2} \Rightarrow 4x + 2 = 6n\pi x + 3\pi x$

$\Rightarrow$ $x = \dfrac{2}{(6n\pi + 3\pi - 4)}$ $n \in I$

or $\sin\left(\dfrac{2x+1}{3x}\right) = \dfrac{3}{4} = \sin\alpha$

$\Rightarrow$ $\dfrac{2x+1}{3x} = n\pi + (-1)^n \alpha$

$\Rightarrow$ $2x + 1 = 3n\pi x + 3(-1)^n\alpha\, x$

$\Rightarrow$ $x = \dfrac{1}{3n\pi - 2 + 3(-1)^n \sin^{-1}\dfrac{3}{4}}$, $n \in I$

Sol.30 $\sin 5x = 16\sin^5 x$

$\Rightarrow$ $\sin 5x = \sin(2x + 3x)$

$= \sin 2x \cos 3x + \cos 2x \sin 3x$

$= 2\sin x \cos x\,(4\cos^3 x - 3\cos x) + (1 - 2\sin^2 x)(3\sin x - 4\sin^3 x)$

$= 2\sin x(1 - \sin^2 x)(4\cos^2 x - 3) + \sin x(1 - 2\sin^2 x)(3 - 4\sin^2 x)$

$= 2\sin x(1 - \sin^2 x)(1 - 4\sin^2 x) + \sin x(1 - 2\sin^2 x)(3 - 4\sin^2 x)$

$= 2\sin x - 10\sin^3 x + 8\sin^5 x + 3\sin x - 10\sin^3 x + 8\sin^5 x$

$= 16\sin^5 x - 20\sin^3 x + 5\sin x$

$\Rightarrow$ $\sin 5x = 16\sin^5 x$ $\Rightarrow$ $-20\sin^3 x + 5\sin x = 0$

$\Rightarrow$ $5\sin x[1 - 4\sin^2 x] = 0$

$\Rightarrow$ $\sin x = 0$ or $\sin^2 x = \left(\dfrac{1}{2}\right)^2$

$\Rightarrow$ $x = n\pi$, $n \in I$ or $x = n\pi \pm \dfrac{\pi}{6}$ $n \in I$

Sol.31 $x\cos^3 y + 3x\cos y\sin^2 y = 14$ ...(i)

$x\sin^3 y + 3x\cos^2 y\sin y = 13$ ...(ii)

divide

$\Rightarrow$ $\dfrac{\cos^3 y + 3\cos y\sin^2 y}{\sin^3 y + 3\cos^2 y\sin y} = \dfrac{14}{13}$

Applying components & dividendo

$\Rightarrow$ $\dfrac{(\cos y + \sin y)^3}{(\cos y - \sin y)^3} = \dfrac{14+13}{14-13}$

$\Rightarrow$ $\left(\dfrac{\cos y + \sin y}{\cos y - \sin y}\right)^3 = 27 = (3)^3$

$\Rightarrow$ $\dfrac{\cos y + \sin y}{\cos y - \sin y} = 3$ $\Rightarrow$ $\dfrac{1 + \tan y}{1 - \tan y} = 3$

$\Rightarrow$ $\tan y = \dfrac{1}{2} = \tan\alpha$

$\Rightarrow$ $y = n\pi + \tan^{-1}\left(\dfrac{1}{2}\right)$, $n \in I$

$\tan y = \dfrac{1}{2}$ $\Rightarrow \sin y = \pm\dfrac{1}{\sqrt{5}}$, $\cos y = \pm\dfrac{2}{\sqrt{5}}$

Case- I: y in Ist quadrant

$\therefore$ $\sin y = \dfrac{1}{\sqrt{5}}$, $\cos y = \dfrac{2}{\sqrt{5}}$

from (i), $\Rightarrow x \times \left(\dfrac{8}{5\sqrt{5}} + 3.\dfrac{2}{\sqrt{5}} \times \dfrac{1}{5}\right) = 14$

$\Rightarrow x = 5\sqrt{5}$

Case - II: y in IIIrd quadrant

$\therefore$ $\sin y = -\dfrac{1}{\sqrt{5}}$, $\cos y = -\dfrac{2}{\sqrt{5}}$

from (i), $\Rightarrow$ $x = -5\sqrt{5}$

Sol.32 $\cot x - 2\sin 2x = 1$ $\qquad \sin x \neq 0$

$\dfrac{\cos x}{\sin x} - 2.2\sin x\cos x = 1$

$\Rightarrow \cos x - 4\sin^2 x\cos x = \sin x$

$\Rightarrow \cos x - 4(1 - \cos^2 x)\cos x = \sin x$

$\Rightarrow \cos x - 4\cos x + 4\cos^3 x = \sin x$

$\Rightarrow 4\cos^3 x - 3\cos x = \sin x$

$\Rightarrow \cos 3x = \sin x$ $\Rightarrow \cos 3x = \cos\left(\dfrac{\pi}{2} - x\right)$

$\Rightarrow 3x = 2n\pi \pm \left(\dfrac{\pi}{2} - x\right)$

$\Rightarrow 3x = 2n\pi + \dfrac{\pi}{2} - x$ or $3x = 2n\pi - \dfrac{\pi}{2} + x$

$\Rightarrow 4x = 2n\pi + \dfrac{\pi}{2}$ or $2x = 2n\pi - \dfrac{\pi}{2}$

$\Rightarrow$ $x = \dfrac{n\pi}{2} + \dfrac{\pi}{8}$ $n \in I$ or $x = n\pi - \dfrac{\pi}{4}$, $n \in I$

or $x = nk + \dfrac{3\pi}{4}$, $n \in I$